Applied Problems 2

Research Methods II
Computational Economics
University of Exeter, 2020

Description:
Please attempt the below two problems. All of the below is written assuming that you will attempt these problems in MATLAB. However, if you wish to attempt these problems in some other matrix based language, this is fine. If you attempt the problems in MATLAB, you will likely be able to use a some of the work that we do together in classes on 07/02/2020 and 14/02/2020 to help in responding to some elements of each problem.

Please send your answers to damian.clarke@protonmail.com by Thursday February 20, 2020 at 23:59. Your answers should consist of the MATLAB functions you write, a README file explaining these functions, and a brief document explaining your response to each of the questions below.

The mark will be assigned in the following way: Does the code for question 1 work in the way it should? (30%); Does the function and code for question 2 work in the way it should? (45%); Clarity of responses in the document submitted (10%); Clarity of documentation to the code including README and comments within the code (15%).

If you have any questions, please write to damian.clarke@protonmail.com. We will discuss a number of tools in class on 14/02/2020 which will likely be useful in resolving these problems. It is not expected that you should be able to respond to these problems before we undertake this discussion in class.

Questions:

1. In this question we will consider utility functions, and utility maximizing consumers. The `fmincon` command is likely useful for resolving this question.

   (a) Generate a function returning utility based on two goods and Cobb-Douglas technology, $u(x_1, x_2) = x_1^{0.5} \cdot x_2^{0.5}$. Assume that the prices for good x_1 and good x_2 are respectively \$1 and \$2. Determine optimal consumption of goods x_1 and x_2 for total incomes, $I \in \{\$50, \$51, \ldots, \$100\}$. This optimal consumption must meet the budget constrant $p_1 x_1 + p_2 x_2 \leq I$. Plot the Engel curves for goods x_1 and x_2.

   (b) Now imagine that there is a limit on the quantity of good x_2, with purchases capped at 20 units per person. Calculate the optimal consumption in this case for all incomes, $I \in \{\$50, \$51, \ldots, \$100\}$, once again plotting the Engel curves, and additionally plotting a figure to document the relationship between income and utility in these cases.

   (c) Repeat question (a), however now where utility is described by a CES utility function: $u(x_1, x_2) = (x_1^{0.6} + x_2^{0.6})^{1/0.6}$.

2. In this question we will return to an econometric application, namely estimation of a probit model by maximum likelihood. To do so, please use the auto.csv dataset from class 2, however now in place of using the categorical variable `mpg` as the outcome variable, generate a new variable y which is equal to 1 if `mpg`> 20, and 0 otherwise. We are interesting in estimating a Probit model, where the outcome variable is y_2 and the dependent variables are price, length and a constant term (the same variables we have used as X in previous examples in this class). *Hint: Using logic in MATLAB is relatively simple. For example, the command* `y = A>0` *will generate a matrix* `y` *which is the same dimension as* `A`*, where each element takes the value of 1 if* `A` *is greater than 0, and 0 otherwise.*

(a) Write a log-likelihood function for a probit model. Remember that in the case of the probit model, the log-likelihood function can be written as:

$$\ell(\beta; y, X) = \sum_{i=1}^{N} \{y_i \ln[\Phi(\beta x_i)] + (1 - y_i) \ln[1 - \Phi(\beta x_i)]\},$$

where Φ is the Normal cumulative density function.

(b) Maximize this log likelihood function to find the parameter vector $\widehat{\beta}$ in the probit model where `y2` is regressed on `price`, `weight` and a constant term. *Hint: For fmincon or fminunc to return a sensible answer, you should ensure that your function returns the negative of the log likelihood function.*

(c) Write a script to estimate the standard errors to this probit model using a bootstrap resample procedure, and your estimation procedure from part (b) of this question. *Hint: we will discuss a script in class on 14/02/2020 which may provide a useful example.*